Cosmological consequences of an inhomogeneous space-time

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February 2, 2008

Abstract

Astrophysical observations provide a picture of the universe as a 4-dim homogeneous and isotropic flat space-time dominated by an unknown form of dark energy. To achieve such a cosmology one has to consider in the early universe an inflationary era able to overcome problems of standard cosmological models.
Here an inhomogeneous model is proposed which allows to obtain a Friedmann-Robertson-Walker behaviour far away from the inhomogeneities and it naturally describes structures formation.
We also obtain that the cosmological term does not prevent structure formation, avoiding a fine tuning problem in initial conditions.
The asymptotic exact solution have been calculated. A simple test with universe age prediction has been performed. A relation between the inhomogeneity, the breaking of time reversal, parity and the matter-antimatter asymmetry is briefly discussed.

keywords:inhomogeneous cosmology, structures formation, matter antimatter asymmetry

PACS number(s) 98.80.Cq, 98.80.Bp, 04.20.Jb

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1 Introduction

Today, thanks to new refined experimental techniques (i.e. neoclassical tests, see [1]) the experimentalists can explore a very large redshift region of the universe. In particular, Supernovae Cosmology Project (SCP) [2], High-Z search Team (HZT) [3] have been able to analyze supernovae data at very high redshift providing an astonishing test able of discriminate between different cosmological models. On the other hand, cosmic microwave surveys [4, 5, 6, 7] have investigated the universe background until the *last scattering surface*, giving significant indications on the geometry of the universe. The overall result is the indication of a cold dark matter universe dominated by an unknown dark energy (Λ-CDM model), which is featured as a spatially flat Friedmann-Robertson-Walker (FRW) manifold from the geometrical point of view.

These great improvements in experimental cosmology represent a challenge for the various theoretical models, which result severely constrained by observational data.

It is a remarkable fact that the best fit model for the dynamical evolution of the universe according to the new experimental data, is a flat FRW cosmology. From the theoretical point of view, the fact that the universe is homogeneous and isotropic to a very high degree of precision rises several problems. The inflationary scenario, thanks to a very fast expansion of early universe which smooths out the inhomogeneities, is the standard way to explain the genesis of such a FRW cosmology (although it is not the unique way, see, for example, [8]).

However, there is still not a commonly accepted model among the various inflationary proposals. For these reasons, it could be useful to study the non-linear evolution of the inhomogeneities unlike the standard case which uses the linearized theory. Here, we propose a relativistic cosmological model which could explain how the inhomogeneities die off during the expansion of the universe and can be developed the structures formation. Our framework is an inhomogeneous 4-dim space-time of phenomenological origin. The study of an inhomogeneous cosmological model has a very long history [10]. However, in the existing literature on this subject it is not easy to make contact with the observations and, in particular, to answer to the question about how the inhomogeneities evolve in time and why the observed universe is so near to a flat FRW. A main attempt to study the cosmological consequences of an inhomogeneous space-time has been developed in 1967 by P.J.E. Peebles [11], deepening the cosmological model based on Tolman-Bondi metric [12]. The author, in particular, obtained interesting results on structures formation starting from a spherically symmetric inhomogeneity in absence of cosmological constant.

Another interesting approach is the so-called "swiss-cheese" cosmological model [13]. This scheme, originated by Lemaitre suggestions was developed by Einstein to study the effects of universe expansion on the solar system [14]. Later on, the properties of this model have been exploited from a cosmological and astrophysical point of view. In such an approach spherical inhomogeneities described by local Schwarzschild or Tolman-Bondi metrics are embedded in a FRW manifold by the means of suitable matching conditions.

Several improvements have been performed in time to this framework [14, 16]. Some considerations on its observational effects have been furnished in [16].
As we will see the model proposed here bears some resemblance with the *swiss-cheese* scheme. However, we will not impose any matching condition and the metric near the inhomogeneity is computed by only looking to at the field equations near the inhomogeneity itself. In this way we will reduce the freedom of the standard swiss-cheese approach. Nevertheless, the quite general result is that, near the inhomogeneity, the metric behaves like a self-similar fluid, thus providing a physical basis to the swiss-cheese models themselves.
The results, derived from the exact non vacuum Einstein equations with a dust source, show that, far away the inhomogeneity the evolution of the universe is almost a standard flat FRW while the size of the inhomogeneity does decrease in time. This behaviour is quite according with the results obtained in [11]. In this context the problem of the structures formation could be accounted in a natural way. The model also reveals an intriguing relation between the smoothing out of the inhomogeneity and the breaking of the time reversal and of the parity.
A further analysis is performed with cosmological constant. The result is a coherent dynamical universe in which structure formation develops in a natural way. Cosmological constant dominates for greater time and induces a de Sitter expansion far away the inhomogeneities.
The work is organized as follows: in Sect.II we propose the model. Sect.III is devoted to study of of the exact asymptotic behaviour. In Sect.IV we provide the analysis in presence of cosmological constant. Sect.V contains a simple test with universe age prediction. Sect.VI is devoted to a summary and to some consideration on the topic.

2 The model

The first problem to construct the model is how to describe an inhomogeneity. On a FRW background, since the metric is homogeneous, the best thing one can do is to study the evolution of the perturbations in the energy density with the linearized Einstein equations. There is an extensive literature on this subject (see for example, [9] and the references therein) and we will not enter in these details. However, the nonlinearities are the most characteristic features of the Einstein equations. Hence, some physical implications of the theory, that manifest themselves in the full Einstein equations, could be lost in the linearized ones. Thus it is interesting to try to use the exact equations.
Following the observations, we will try to describe a flat FRW model with an inhomogeneity. As source we will take simply a dust energy-momentum tensor:

$$T_{\mu\nu} = \rho u_\mu u_\nu \,, \tag{1}$$

where the energy density ρ does depend on the radial coordinate too: $\rho = \rho(t, r)$.
We will take the metric in the form:

$$\begin{aligned} g_{\mu\nu} dx^\mu dx^\nu &= ds^2 = dt^2 - \frac{2K}{A} dt dr - A^2 \left[dr^2 + r^2 d\Omega \right], \\ A &= A(t, r), \;\; K = const > 0 \,. \end{aligned} \tag{2}$$

We make this ansätz to careful describe the inhomogeneity. The $r-$dependence of the scale factor A takes into account the radial deformations of the Σ_t ($t = const$) hypersurfaces which slice space-time in FRW metric. The nondiagonal term in the metric $g_{tr} = -2K/A$ gives a radial component to the dust velocity field:

$$g_{tr} \neq 0 \Rightarrow u_r \neq 0. \tag{3}$$

The particular form chosen for g_{tr} will allow to find, in the limit $A \gg 1$, solutions approaching to a FRW metric and, in such a case, t becomes the physical co-moving time. Of course, it is always possible to diagonalyze the metric (2) with a suitable coordinates transformation and in such a case one obtains a slight generalization of the ansätz considered in [11]. However, in the present case, the diagonalized form of the new timelike coordinate[1] cannot be related anymore in a direct way to the cosmic time so that, in the new coordinates system, the physical interpretation is less transparent.
We could choose a different form for g_{tr}, but, among the various natural choices (such as $g_{tr} \sim \dfrac{1}{A^n}$ with $n > 1$, or $g_{tr} \sim \exp(-A)$), this simplifies somehow the calculations. Finally, the constant K, that, for sake of simplicity could be put equal to one, is useful to recognize the role of g_{tr}.
Now, the explicit expressions of the components of the Einstein equations are not particulary expressive. Thus, we write down only the main equation we have to solve:

$$G_{33} = 0 \Rightarrow$$

$$\begin{aligned} rA^3 \left(5K^2 + A^4\right) (\partial_t A)^2 + rK \left(A^4 - K^2\right) \partial_t A \partial_r A - K^3 2A \partial_t A + \\ + rA \left(A^4 - K^2\right) (\partial_r A)^2 - A^2 \left(3K^2 + A^4\right) \partial_r A + 2rA^4 \left(K^2 + A^4\right) \partial_t^2 A + \\ - KrA \left(K^2 + A^4\right) \partial_{tr}^2 A - rA^2 \left(K^2 + A^4\right) \partial_r^2 A = 0 \,, \end{aligned} \tag{4}$$

the other equations (related to the other components of Einstein tensor) determine simply the explicit form of energy-momentum tensor once the scale-factor A has been calculated by (4).

[1] Roughly speaking when we diagonalyze metric (2) it appears a non trivial coefficient in front of the $(dt')^2$.

Eq.(4) is highly nonlinear and it cannot be solved by separation of variables. However, its asymptotic behaviour allows to clarify some important physical aspects of the problem. In fact, this equation tells us that it is natural to separate the evolution for big r (i.e. far away the inhomogeneity) from the evolution for small r (i.e. on the "top" of the inhomogeneity). It is clear that for $r \to \infty$ the terms multiplied by r will dominate, while for $r \to 0$ the terms without r will dominate.
As an important remark we stress that Eq.(4) is not scale invariant. Namely, if A is a solution then λA (with constant λ) cannot be a solution. This means that due to the g_{tr} coefficient there is a characteristic scale in the dynamics of A which we will call $A_{Crit} \sim K^{\frac{1}{2}}$.
Another interesting consequence of the model, related to the nondiagonal term in the metric, is that the dynamics is not invariant under the time reversal T: $t \to -t$ and under the parity P: $r \to -r$ (although it is invariant under TP). This fact could be very interesting if t would be the physical comoving time, but t is not. However, if one finds a FRW-like solution, in this limit, t is the physical comoving time and one is allowed to relate, as we will see, this breaking of T and P to the evolution of the inhomogeneity.
Previous considerations indicates some differences between our model and swiss-cheese ones. In swiss-cheese models one imposes by hand the matching conditions needed to join the internal inhomogeneous space-time (such a Schwarzschild or a self-similar fluid) to the external FRW space-time. This approach results in a non-smooth metric at the embedding hyper-surface. On the other side in our scheme it is possible to study the general behaviour of the space-time near and far-away the inhomogeneity without any matching condition. As a consequence this approach allows to define observable quantities, i.e. the age of universe, in term of parameters characterizing the intrinsic size of the inhomogeneity.
It is worth to note that it is impossible to shed light on the relation between the breaking of T and P and the inhomogeneity in the standard swiss-cheese models.

3 Asymptotic behaviour

As remarked in Sec.1, because of the complexity of Eq.(4), the model solution will be studied only in the asymptotic cases. We start by analyzing the behaviour far away from the inhomogeneity, as second case we propose the near limit.

3.1 Far away the inhomogeneity: a FRW-like evolution

The model behaviour far away the inhomogeneity can be achieved in the limit $r \to \infty$. In this case, Eq.(4) becomes:

$$\begin{aligned} & A^3\left(5K^2+A^4\right)(\partial_t A)^2 + K\left(A^4-K^2\right)\partial_t A \partial_r A + \\ & + A\left(A^4-K^2\right)(\partial_r A)^2 + 2A^4\left(K^2+A^4\right)\partial_t^2 A + \end{aligned} \quad (5)$$

$$-KA\left(K^2+A^4\right)\partial^2_{tr}A - A^2\left(K^2+A^4\right)\partial^2_r A = 0\,.$$

The model, to have the physical interpretation of an universe with an inhomogeneity only at small r, must admit solutions that do not depend on r in the limit $r \to \infty$. Eq.(5) has a solution depending only on t. Indeed, writing $A(t,r) = A(t)$ it becomes:

$$\left(5K^2+A^4\right)(\partial_t A)^2 = -2A\left(K^2+A^4\right)\partial^2_t A\,. \tag{6}$$

As a consequence A is implicitly given by the following expression:

$$\begin{aligned} I_1 t + I_2 &= \int^{A'}\sqrt{\frac{x^5}{1+x^4}}dx\,, \\ A' &= \frac{A}{K^{\frac{1}{2}}} \quad I_i = const\,, \end{aligned} \tag{7}$$

where I_i are integration constants.
It is possible to verify that for great values of time such a solution reduces to $A(t) \sim t^{2/3}$. This result is intriguing, according with a FRW evolution for dust matter as source.
As a consequence, we are allowed to interpret t as the asymptotic comoving time, without loss of generality. This fact will be relevant in Sect.V.
Moreover such a result is quite natural since we have not imposed any matching condition unlike swiss-cheese models.

3.2 On the top of the inhomogeneity: a decreasing scale factor

Now, we will study the dynamics near the inhomogeneity, i.e. in the limit $r \to 0$. In this case Eq.(4) becomes:

$$-K^3 2\partial_t A - A\left(3K^2+A^4\right)\partial_r A = 0\,. \tag{8}$$

It is important to stress here that, with $g_{tr} = 0$ (i.e. $K = 0$), the above equation would become trivial. Then, the importance of the role of the inhomogeneity manifests itself for small r. Eq.(8) has no $r-$independent solution and, moreover, cannot be solved by separation of variables. Nevertheless, it is possible find an interesting exact solution. As an intermediate step, let us study firstly the two extreme cases: *i)* $A \gg K^{\frac{1}{2}}$, *ii)*$A \ll K^{\frac{1}{2}}$.

i) $A \gg K^{\frac{1}{2}}$; In this case the Eq.(8) reads:

$$-K^3 2\partial_t A - A^5\partial_r A = 0\,, \tag{9}$$

and it can be now solved by separation of variables. Taking $A(t,r) = \overline{a}(t)\overline{b}(r)$, we obtain the following solution:

$$\begin{aligned} \overline{a}(t) &= K^{\frac{1}{2}}\left(d_1 + \frac{5}{2}\sigma\frac{t}{K^{\frac{1}{2}}}\right)^{-\frac{1}{5}} \\ \overline{b}(r) &= (d_2 + 5\sigma r)^{\frac{1}{5}}\,, \end{aligned} \tag{10}$$

where the d_i are integration constants and σ is the separation constant. We get the interesting result that, on the top of the inhomogeneity the scale factor is a decreasing function of t, the part of the solution depending by r approaches to constant for $r \to 0$.

ii) $A \ll K^{\frac{1}{2}}$; In this other case Eq.(8) becomes:

$$-K2\partial_t A - 3A\partial_r A = 0\,. \tag{11}$$

Also in this case it is possible to separate the variables. By taking again $A(t,r) = \overline{a}(t)\overline{b}(r)$, one shows that:

$$\begin{aligned} \overline{a}(t) &= K^{\frac{1}{2}}\left(d_3 + \frac{1}{2}\sigma'\frac{t}{K^{\frac{1}{2}}}\right)^{-1} \\ \overline{b}(r) &= d_4 + \frac{1}{3}\sigma' r\,, \end{aligned} \tag{12}$$

where again the d_i are integration constants and σ' is the separation constant. In this case also one obtains that on the top of the inhomogeneity the scale factor, and then the size of the inhomogeneity, is a decreasing function of t, while the radial part of the solution tends to a constant value.

It is useful to note that in both cases, if A depends on r^α (with $\alpha > 0$) then A depends on $t^{-\alpha}$. Hence, it is natural to try to find a solution of the full equation (8) in the form $A = A(r/t)$. By substituting this ansätz in the (8), one immediately gets the following implicit expression of A as function of r/t:

$$\frac{A}{K^{\frac{1}{2}}}\left(3 + \left(\frac{A}{K^{\frac{1}{2}}}\right)^4\right) = 2\frac{K^{\frac{1}{2}}r}{t}\,. \tag{13}$$

A few remarks on this solution are in order.
First, A is not factorized and has a true singularity for $r = 0$ (there the Ricci scalar diverges). On the other hand, it interpolates between the two extreme behaviors: for large t one has $A \sim (r/t)$ while for small t one has $A \sim (r/t)^{\frac{1}{5}}$. Moreover, A is always a decreasing function of t, this implies that the size of the inhomogeneity does decrease with time and at the same time the matter density increases.
It is important to stress that the term in the equation that implies such an effect, i.e. $-K^3 2A\partial_t A$, is strictly related to the non-diagonal term: in fact, with $g_{tr} = 0$ this term would be zero.
It has to be remarked that, if one uses the linearized Einstein equations and treats the non-diagonal term as a small perturbation, then this effect disappears, since it is of the third order in K. The decreasing of the size of the inhomogeneity could explain in a natural way the structures formation (see also [11]). In fact, a decreasing scale factor implies an increasing density ρ. Thus, one would expect that, when the density reaches a critical

value ρ_{Crit} [2]. (that depends on the actual cosmological structure we are considering, such as galaxies, cluster of galaxies, etc.), then the structure begins the formation decoupling from the outside universe that evolves as in the usual FRW models. Of course, when $\rho > \rho_{Crit}$, one has to consider the contribution of the pressure to the energy-momentum tensor.

Since in this case the integral curves of ∂_t are not geodesic, this model could suggest an interpretation of the observed acceleration of the galaxies without the introduction of an extra scalar field. In fact, if the galaxies would follow the integral line of ∂_t, then these would accelerate with respect to each others. Thus the observed negative deceleration parameter could be simply explained as an effect of geometrical origin.

Eventually, we want to comment the symmetry breaking. In this model, the smoothing out of the inhomogeneity and the breaking of T and P are strictly related. Both phenomena have the same physical origin: the non-diagonal term. In other words, one cannot obtain the first without the second and viceversa. In the standard perturbative approaches to the study of the cosmological anisotropies, one cannot see this relation because the FRW background is T and P invariant. Only the full theory can reveal this interesting connection. This could have important phenomenological consequences for the matter-antimatter asymmetry. In fact, although almost all the calculations of particles creation by time-dependent gravitational field predict an equal number of particles and anti-particles, it is well known that a violation of the homogeneity or of the parity can break this symmetry (see, for example, [17]) (even in the paper of Parker [18], one of the first work on the subject, it is stressed that the fact that the particles and antiparticles are created in pair is related to the homogeneity). Moreover, it has been shown by Klinkhamer [19] that even gauge theory on flat background, but with a nontrivial topology, can have CPT breaking and in our case, since we have a singularity for $r = 0$, the topology of the spacelike slices is $R^3 \setminus \{0\}$. So, this kind of effect could have important consequences in the study of the matter-antimatter asymmetry problem.

4 The model with cosmological constant

In this section we analyze the model in presence of cosmological constant.

The field equations, as it is well known, read:

$$G_{\mu\nu} = T_{\mu\nu} + \Lambda g_{\mu\nu}. \tag{14}$$

By these, inserting the metric described in relation (2), we obtain the 3-3 component which determines the cosmological dynamics:

$$G_{33} = \Lambda g_{33} \Rightarrow$$

[2]We stress that with ρ_{Crit} we are referring to the critical matter density needed to allow gravity to form structures. Obviously this definition must be carefully distinct from the analogous expression with which is characterized the energy density amount able to provide a spatially flat universe

$$
\begin{aligned}
&rA^3\left(5K^2+A^4\right)(\partial_t A)^2 + rK\left(A^4-K^2\right)\partial_t A\partial_r A + 2rA^4\left(K^2+A^4\right)\partial_t^2 A + \\
&+rA\left(A^4-K^2\right)(\partial_r A)^2 - A^2\left(3K^2+A^4\right)\partial_r A - K^3 2A\partial_t A + \\
&-KrA\left(K^2+A^4\right)\partial_{tr}^2 A - rA^2\left(K^2+A^4\right)\partial_r^2 A = -\Lambda rA\left(A^4+K^2\right)^2 .
\end{aligned} \tag{15}
$$

Now, we can study, again, the model far away and near the inhomogeneity.

In the first case we consider $r \to \infty$ and a r-independent solution, so that $\partial_r A \to 0$. The result is a simple generalization of the previous case:

$$
2K^2A^3\partial_t^2 A + 5K^2A^2(\partial_t A)^2 + A^6(\partial_t A)^2 + 2A^7\partial_t^2 A = -\Lambda(A^4+K^2)^2 \tag{16}
$$

which can be rearranged in a more expressive form

$$
-\Lambda(1+\frac{K^2}{A^4})^2 = 2K^2\frac{\partial_t^2 A}{A^5} + 5K^2\frac{(\partial_t A)^2}{A^6} + \frac{(\partial_t A)^2}{A^2} + 2\frac{\partial_t^2 A}{A} . \tag{17}
$$

It is clear that, for great values of A, this relation becomes the pressure Friedmann equation for the standard cosmology (i-i component of Einstein equation in FRW-metric) in presence of cosmological constant [1, 20]; we recall that we have assumed a pressureless matter fluid, so that we obtain:

$$
-\Lambda = \frac{(\partial_t A)^2}{A^2} + 2\frac{\partial_t^2 A}{A} . \tag{18}
$$

It can be shown that Eq.(17) has a solution which, for $t \to \infty$, agrees with the requested de Sitter expansion for a FRW spatially flat model with cosmological constant. In other words, we achieved an asymptotically de Sitter behaviour as a natural effect of the model for great values of r.

Let us we study our model near the inhomogeneity (i.e. in the limit $(r \to 0)$). Eq.(15) gives:

$$
\frac{1}{A^4}\left(3A^2K^2\partial_t A + 2AK^3\partial_t A + A^6\partial_r A\right) = \Lambda Ar . \tag{19}
$$

For $r \to 0$ the right term fall down and (19) becomes the same as in the case without cosmological constant

$$
(3A+2K)\,K^2\partial_t A + A^5\partial_r A = 0 . \tag{20}
$$

Thus, a cosmological constant term inside this inhomogeneous cosmological model does not destroy the inhomogeneity and, more important, this term does not influence the evolution of such an inhomogeneity. Such a behaviour is not trivial. Unlike in the standard theory (see [9] and references therein) in which the Λ-term can prevent structures

formation, in this model structure formation will be preserved in presence of cosmological constant. Furthermore, it is not required any sort of fine tuning between matter and cosmological component to allow to cosmological perturbations of determine today observed structures.

5 A simple test

To simply test our model, we have checked its capability of providing a significant prediction of the age of the universe.
To perform this test we use the exact solution obtained far away the inhomogeneity by which is significant the use of t as comoving time.
The scale factor was determined by the expression:

$$I_1 t + I_2 = \int^{A'} \sqrt{\frac{x^5}{1+x^4}} dx \,, \tag{21}$$

in this limit.
In this model, the age of the universe is directly related to the ratio between the actual size of the universe and the characteristic size of the inhomogeneity ($K^{1/2}$).
It is always possible to assume $I_2 = 0$, thus Eq.(21) can be written as:

$$t = \frac{1}{I_1} \int^{A'} \sqrt{\frac{x^5}{1+x^4}} dx \,. \tag{22}$$

It is worth to note that the predicted age of the universe results slightly smaller than the estimate for standard FRW and swiss-cheese models(in particular, the greater is the size of the inhomogeneity the smaller is the predicted age [3]). We can compute the value of A' at today by taking the current time deduced by observations. We will take I_1 as the measure unit fixing it to one year.
We consider for t the value provided by the last WMAP observations, about $13.7^{+0.2}_{-0.2} Gyr$ at 1-σ level [7].
By this calculation we obtain an estimate of $A'_{today} \simeq 7500000$. Remembering the definition of A' as $A/K^{1/2}$ we can find a relation between today scale factor and $K^{1/2}$:

$$A_{today} \sim 7.5 \cdot 10^6 K^{1/2}, \tag{23}$$

which can be compared with the estimate of the ratio between scale factor and structures. If we want to refer to the 1σ range of WMAP universe age we obtain an estimate for A'

[3] The smaller value of age prediction for our model descends by the presence of the sum in the denominator of right member of (22). In the standard case this term would be simply x^4.

to be comprised between $]\,7430000, 7580000\,[$.
As a second case we can perform the same calculation considering now the recombination value of cosmic time (i.e. the instant in which matter has become transparent to radiation). So we can provide a theoretical estimate by our model between inhomogeneity size (which acts as a seed of structures) and scale factor at this time.
Obviously, we are supposing to consider the last scattering surface as boundary of the causal universe. As a consequence, the estimate of A size corresponds to Hubble radius estimate at a certain time. In this sense it is possible to use again the far away limit and thus Eq.(22) for the age evaluation.
Now, t_{recomb} is attested to the value of 379^{+8}_{-7}kyr as deduced by WMAP data [7]. By the numerical integration of Eq.(22):

$$A'_{recomb} = A_{recomb}/K^{1/2} \sim 6863 \tag{24}$$

or more exhaustively, running into the 1-σ errors for t_{recomb} we have $A' \in \,]\,6780, 6960\,[$.
It is evident that the inhomogeneity decreased in time.

6 Summary

In this paper, we studied a model for the evolution of the universe in presence of a spherically symmetric inhomogeneity. This inhomogeneity has been introduced in a FRW-like metric by a nontrivial dependence on r of the scale factor and by a non-diagonal element in the metric. A similar approach has been already considered in [11] considering a diagonal metric in absence of cosmological constant. This non-diagonal element does introduce in the model a characteristic scale A_{Crit} ($\sim K^{1/2}$) for A and a breaking of the time reversal T and of parity P symmetries, which are absent in the homogeneous cosmological models. We consider, as source, a dust energy-momentum tensor.
Then we solved the exact Einstein equation separately for large r and small r. We showed that for r and t large a FRW-like behaviour is recovered. On the other side for small r, thanks to the non-diagonal term, universe dynamics is strongly affected by inhomogeneities, in particular the scale factor does decrease with time. The decreasing in size of the inhomogeneity could give a natural mechanism for the structures formation without the need of an inflationary era. Besides, this model shows an interesting connection between the mechanism of structures formation and the breaking of the time reversal (the, so called, "arrow of time") and of the parity, in other words they have the same physical origin. This result has been obtained without the inclusion of the cosmological constant or of any other kind of negative pressure.
To complete the study we have proposed an analysis of the model in relation to cosmological constant. It is obtained that a Λ-term, unlike in the standard linearized theory, does not affect inhomogeneity dynamics preserving structures formation. On the contrary such a term induces a significant influence on the far away evolution driving towards a de Sitter-like expansion.

A simple test on the capability of the age prediction has been performed, obtaining an estimate of scale factor ratio versus inhomogeneity characteristic size. This calculation has been performed both for today and recombination values of universe time.
Of course, there are many questions to be clarified. It would be important to know how general are the characteristic features of the model and how it depends on our technical assumptions (such as our choice $g_{tr} \sim \frac{1}{A}$). Besides, one should have a more rigorous analysis of the model to test the roboustness and the stability of the results. Alternatively, it would be preferred to have numerical solutions able to fit the proposed asymptotic behaviours (some preliminar calculations are quite encouraging).
The last interesting question we want to mention here, is the study of the QFT on a non P-invariant and non T-invariant background. In fact, there could be observable consequences for the explanation of the matter-antimatter asymmetry in the study of the creation of particles by the time evolution of the universe.

Acknowledgement

F. Canfora and A. Troisi are very grateful to the invaluable suggestions and the encouragements provided to them by Prof. G. Vilasi and Dr. S. Capozziello during their work. The authors want to thank also their friends V. F. Cardone and S. Carloni for the useful discussions on the topics.

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